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ABSTRACT

We have solved the Dirac equation with an anomalous moment Pauli-coupling exactly for a constant magnetic field and derived the general relativistic formulas for phase changes due to translational motion and spin rotations. We also give transmission and reflection coefficients, spin rotation for tunnelling and barrier penetration. For ultrarelativistic particles the spin rotation angle on the path of length L is equal to $(2\mu B L/\hbar c)[1+m^2c^4/(E^2-\mu^2B^2)]$.

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I. Introduction

We study in this paper the passage of relativistic neutral spin 1/2-particles with an anomalous magnetic moment through magnetic fields. The most important examples are relativistic neutrons and neutrinos. The latter could have a small, but in some applications significant, magnetic moment. This investigation generalizes a recent work on the spin rotation and reflection and transmission properties of nonrelativistic spin 1/2-magnetic dipoles through magnetic fields[1]. It is interesting to see how highly relativistic particles and massless particles which have no nonrelativistic limit behave in this respect, and whether qualititively different and new phenomena occur for fast particles. To our knowledge the reflection and transmission coefficients and the related spin rotation formulas for the general case obtained here are new.

A small magnetic moment for the neutrino is known to lead to bound states with another particle[2]. Furthermore, a small magnetic moment for the neutrino of the order of a few times $10^{-9}\mu_0$ reproduces the neutral weak current scattering $e+\nu_e \to e+\nu_e$ as experimentally observed, or as given by Weinberg-Salam electroweak model which thus sets an upper limit to the neutrino magnetic moment[3,4].

Recently magnetic moments of the order of $10^{-10}\mu_o$ have been invoked to explain the solar neutrino puzzle[5-8]. Our results should have some bearing for this explanation[9] as well as for neutron experiments with ultracold neutrons at very high magnetic fields.

There are two ways in which the flux of the spin 1/2 particles coming from the interior of the Sun can change in the magnetic field of the Sun: by rotation of spin or helicity oscillations, or by attenuation. In order to investigate these effects quantitativesy we have also studied the barrier penetration of spin 1/2 particles through a field of length a.

In Section II we derive the four independent solutions of the Dirac equation for neutral spin 1/2 particle in a constant

magnetic field. In Section III we study general positive energy eigenstates and identify the part of the wave function which describes the translational and the part that describes the internal motion and determine the frequency of spin precession in the Laboratory frame.

In Section IV we consider the transmission through a region of constant magnetic field of a general positive energy incoming stationary wave. We determine the transmission and reflection coefficients in the case of real and imaginary wave vectors inside the fièld. In Section V the composition law for tunnelling is given. The conditions of minima and maxima of reflections and transmissions are interpreted on the basis of this composition law.

At the end we determine rotation angles and transmission coefficients in the weak field limit, in ultrarelativistic and nonrelativistic cases. The ultrarelativistic case is relevant for the study of the possibility to explain the solar neutrino puzzle through helicity oscillations or through the reflection of the neutrino beam in the magnetic field.

II. Neutral Dirac particle with Pauli coupling

We shall study the equation

$$(\hbar\gamma^\mu i\partial_\mu - mc)\psi + \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu}\psi = 0 \tag{1}$$

where ψ is the 4-component spinor, μ the anomalous magnetic moment of the particle and $F_{\mu\nu}$ an external electromagnetic field. With $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ the coupling term is

$$\frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu} = \mu(\vec{\Sigma}\cdot\vec{B} - i\vec{\alpha}\cdot\vec{E})$$

where $\vec{\Sigma}$ and $\vec{\alpha}$ are given by

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}, \quad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{2}$$

with $\vec{\sigma}$ as the Pauli matrices.

The Hamiltonian corresponding to eq.(1) is

$$\hat{H} = c\vec{\alpha}\cdot\vec{p} + \beta mc^2 - \mu\beta(\vec{\Sigma}\cdot\vec{B} - i\vec{\alpha}\cdot\vec{E}) = c\vec{\alpha}\cdot\vec{\pi} + \beta mc^2 - \mu\beta\vec{\Sigma}\cdot\vec{B} \tag{3}$$

with

$$\vec{\pi} = \vec{p} + i\mu\beta\frac{\vec{E}}{c}$$

We shall consider constant electromagnetic fields $\vec{E}$ and $\vec{B}$ and look for stationary solutions of the form

$$\psi = e^{i(\vec{p}\cdot\vec{r} - Et)/\hbar}\begin{pmatrix} \varphi \\ \chi \end{pmatrix} \tag{4}$$

This ansatz leads to the coupled equations

$$\begin{aligned} (E - mc^2 + \mu\vec{\sigma}\cdot\vec{B})\varphi &= c\,\vec{\sigma}\cdot\vec{\pi}\,\chi \\ (E + mc^2 - \mu\vec{\sigma}\cdot\vec{B})\chi &= c\,\vec{\sigma}\cdot\vec{\pi}\,\varphi \end{aligned} \tag{5}$$

Eliminating χ from the first equation of (5).

$$\chi = \frac{1}{c\pi^2}[(\vec{\sigma}\cdot\vec{\pi})(E - mc^2) + \mu(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})]\varphi$$

and inserting into the second equation of (5), we obtain

$$c\,\vec{\sigma}\cdot\vec{\pi}\,\varphi = (E + mc^2 - \mu\vec{\sigma}\cdot\vec{B})\frac{1}{c\pi^2}[(\vec{\sigma}\cdot\vec{\pi})(E - mc^2) + \mu(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})]\varphi$$

or

$$c^2\pi^2\varphi = [(\vec{\sigma}\cdot\vec{\pi})(E + mc^2) - \mu(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})]\frac{1}{\pi^2}[(\vec{\sigma}\cdot\vec{\pi})(E - mc^2) + \mu(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})]\varphi$$

Since $\vec{\pi}^2 = (\vec{p} + i\frac{\mu}{c}\beta\vec{E})^2$ commutes with the other factors around it, we have

$$\begin{aligned} c^2\pi^4\varphi = [&\vec{\pi}^2(E - m^2c^4) - \mu(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})(\vec{\sigma}\cdot\vec{\pi})(E - mc^2) \\ &+ \mu(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})(E + mc^2) - \mu^2(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})]\varphi \end{aligned}$$

The identity $(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B}) = \vec{\pi}\cdot\vec{B}+i\,\vec{\sigma}\,(\vec{\pi}\times\vec{B})$ gives

$$(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})(\vec{\sigma}\cdot\vec{\pi}) = -\pi^2(\vec{\sigma}\cdot\vec{B})+2(\vec{\pi}\cdot\vec{B})(\vec{\sigma}\cdot\vec{\pi})$$

and

$$(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{B})(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{B}) = -\pi^2B^2+2(\vec{\pi}\cdot\vec{B})^2+2i(\vec{\pi}\cdot\vec{B})\,\vec{\sigma}(\vec{\pi}\times\vec{B})$$

so that

$$\pi^4\varphi = \{\pi^2(E^2-m^2c^4)+\mu\pi^2 2E(\vec{\sigma}\cdot\vec{B})-2\mu(E-mc^2)(\vec{\pi}\cdot\vec{B})(\vec{\sigma}\cdot\vec{\pi})$$
$$+\mu^2\pi^2B^2-2\mu^2(\vec{\pi}\cdot\vec{B})^2-2i\mu^2(\vec{\pi}\cdot\vec{B})\,\vec{\sigma}\,(\vec{\pi}\times\vec{B})\}\varphi \qquad (6)$$

<u>Special Cases</u> that can be easily solved are:

a) Constant Electric Field alone.

$$\pi^2\varphi = (E^2-m^2c^4)\varphi$$

b) Perpendicular electric and magnetic fields:

$$\vec{E}\perp\vec{B} \qquad \vec{\pi}\cdot\vec{B} = \vec{p}\cdot\vec{B}$$

But in this paper we shall consider the case of constant magnetic fields only. In that case we have $\vec{\pi} = \vec{p}$.

III. <u>Motion of the Dipole in a constant Magnetic Field</u>

We shall consider particles moving perpendicular to the magnetic field $\vec{B}$.

We thus have

$$\vec{p}\cdot\vec{B} = 0; \qquad \vec{\pi} = \vec{p} \qquad (7)$$

and (6) reduces to

$$p^4\varphi = [p^2(E^2-m^2c^4)+p^2\mu(E-mc^2)(\vec{\sigma}\cdot\vec{B})+\mu p^2(\vec{\sigma}\cdot\vec{B})\,(E+mc^2)$$
$$+p^2\mu^2B^2]\varphi$$

or, with $p^2\varphi = \phi$, we have the second order equation

$$p^2\phi = [(E^2-m^2c^4)+2\mu(\vec{\sigma}\cdot\vec{B})\cdot E+\mu^2B^2]\phi \qquad (8)$$

We can now choose ϕ to be the eigenstates of $\vec{\sigma}\cdot\vec{B}$

$$(\vec{\sigma}\cdot\vec{B})\phi^{\pm} = \pm B\phi^{\pm} \qquad (9)$$

Then

$$p^{\pm 2}\phi^{\pm} = [E^2-m^2c^4\pm 2\mu BE+\mu^2B^2]\,\phi^{\pm} \qquad (10)$$

The comply with the nonrelativistic usage we denote $p^{\pm}$ also as p' and p":

$$c^2p'^2 = (E+\mu B)^2-m^2c^4$$
$$c^2p''^2 = (E-\mu B)^2-m^2c^4 \qquad (11)$$

and we define the momentum of the free particle corresponding to the energy E

$$c^2p^2 = E^2 - m^2c^4 \qquad (12)$$

<u>Eigenspinors in the Magnetic field</u>

With the help of the basic spinors $\phi^{\pm}$ given in (9) we now construct four independent solutions of the Dirac equation (1) in a constant magnetic field. The method consists in writing the solution as $U = N\begin{pmatrix}\phi\\ A(\vec{\sigma}\cdot\vec{p})\phi\end{pmatrix}$, and determining A from eq.(5) and N by normalization condition $U^+U = 1$. In this way using (10) we obtain two positive energy solutions:

$$U_1 = \left[\frac{E+mc^2+\mu B}{2(E+\mu B)}\right]^{1/2} \begin{pmatrix} \phi^+ \\ \frac{c(\vec{\sigma}\cdot\vec{p}\,')}{E+mc^2+\mu B}\phi^+ \end{pmatrix} = \frac{1}{\sqrt{2(E+\mu B)e'}} \begin{pmatrix} e'\phi^+ \\ (\vec{\sigma}\cdot\vec{p}\,')\phi^+ \end{pmatrix}$$

$$U_2 = \left[\frac{E+mc^2-\mu B}{2(E-\mu B)}\right]^{1/2} \begin{pmatrix} \phi^- \\ \frac{c(\vec{\sigma}\cdot\vec{p}\,'')}{E+mc^2-\mu B}\phi^- \end{pmatrix} = \frac{1}{\sqrt{2(E-\mu B)e''}} \begin{pmatrix} e''\phi^- \\ (\vec{\sigma}\cdot\vec{p}\,'')\phi^- \end{pmatrix}$$

$$E = \sqrt{p'^2c^2 + m^2c^4} - \mu B, \qquad E = \sqrt{p''^2c^2 + m^2c^4} + \mu B \tag{13}$$

and two negative energy solutions

$$U_3 = \left[\frac{E+mc^2+\mu B}{2(E+\mu B)}\right]^{1/2} \begin{pmatrix} -\frac{c(\vec{\sigma}\cdot\vec{p}\,')}{E+mc^2+\mu B}\phi^- \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2(E+\mu B)e'}} \begin{pmatrix} -c(\vec{\sigma}\cdot\vec{p}\,')\phi^- \\ e'\phi^- \end{pmatrix}$$

$$U_4 = \left[\frac{E+mc^2-\mu B}{2(E-\mu B)}\right]^{1/2} \begin{pmatrix} -\frac{c(\vec{\sigma}\cdot\vec{p}\,'')}{E+mc^2-\mu B}\phi^+ \\ \phi^+ \end{pmatrix} = \frac{1}{\sqrt{2(E-\mu B)e''}} \begin{pmatrix} -c(\vec{\sigma}\cdot\vec{p}\,'')\phi^+ \\ e''\phi^+ \end{pmatrix}$$

$$E = -\sqrt{p'^2c^2 + m^2c^4} - \mu B, \qquad E = -\sqrt{p''^2c^2 + m^2c^4} + \mu B \tag{14}$$

where

$$e' = E+mc^2+\mu B$$

$$e'' = E+mc^2-\mu B$$

$$e = E+mc^2 \tag{15}$$

For B=0, $cp'=cp''=cp=(E^2-m^2c^4)^{1/2}$, $e'=e''=e$ and the solutions go over to the free particle solutions of the Dirac equation.

When $\vec{p}$ is in x-direction and we choose $\phi^+ = \binom{1}{0}$, $\phi^- = \binom{0}{1}$ we have more specifically

$$U_1(p',B) = \frac{1}{\sqrt{2(E+\mu B)e'}} \begin{pmatrix} e' \\ 0 \\ 0 \\ cp' \end{pmatrix}, \quad U_2(p'',B) = \frac{1}{\sqrt{2(E-\mu B)e''}} \begin{pmatrix} 0 \\ e'' \\ cp'' \\ 0 \end{pmatrix} \tag{16}$$

III. <u>Spin rotations in the Laboratory frame</u>

The general positive energy eigenstate of $\hat{H}$ which corresponds to the motion of the particle in the positive x-direction has therefore the form:

$$\phi_{II}(x,t) = e^{-iEt/\hbar} \cdot \phi_{II}(x) \tag{17}$$

where $\phi_{II}(x)$ is a bispinor which belongs to the subspace spanned by two positive energy eigenspinors U_1 and U_2:

$$\phi_{II}(x) = e^{ik'x}\alpha' U_1(k';B) + e^{ik''x}\beta'' U_2(k'';B) \tag{18}$$ *

For further purposes it is convenient to associate with each four component spinor $\phi_{II}(x)$ a two component spinor:

$$\hat{\phi}_{II}(x) = \begin{pmatrix} \alpha' e^{ik'x} \\ \beta'' e^{ik''x} \end{pmatrix} \tag{19}$$

representing the components of $\phi_{II}(x)$ in the U_1-U_2 basis.

Both of the above given forms of the eigenstate have the disadvantage that they do not expose explicitly neither the symmetry groups of the Hamiltonian nor they allow to identify the part of the function which describes the translational and the part which describes the internal motion.

The symmetry group in a constant electromagnetic field is in general a six dimensional subgoup of the Poincaré group consisting of four translations, one rotation and one boost along

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* **We shall drop the Volume normalization factor $1/\sqrt{V}$ in front of ϕ because it does not affect observable quantities.**

the field. In our case it reduces to one translation, one rotation and one pure Lorentz transformation.

In order to make the existence of translation and rotation groups explicit we associate with the evolution of a spinor wave (19) in the positive x-direction between two points x_1 and x_2 the transformation matrix $\hat{W}_{x_1 \to x_2}$:

$$\hat{\Phi}_{II}(x_2) = \hat{W}_{x_1 \to x_2} \hat{\Phi}_{II}(x_1) \tag{20}$$

It follows that:

$$\hat{W}_{x_1 \to x_2} = \begin{pmatrix} e^{ik'(x_2-x_1)} & o \\ o & e^{ik''(x_2-x_1)} \end{pmatrix} \tag{21}$$

$\hat{W}_{x_1 \to x_2}$ has to be the product of the one-dimensional representation $D_{\bar{k}}(x_2 - x_1)$ of the translation for x_2-x_1 (with $\bar{k}$ to be determined) and of a spinor representation of the rotation around the z-axis for certain angle φ which also has to be determined.

$$D^{o\frac{1}{2}} = \begin{pmatrix} e^{-i\varphi/2} & o \\ o & e^{i\varphi/2} \end{pmatrix} \tag{22}$$

It turns out that the equality

$$\hat{W}_{x_1-x_2} = \begin{pmatrix} e^{ik'(x_2-x_1)} & o \\ o & e^{ik''(x_2-x_1)} \end{pmatrix} = e^{i\bar{k}(x_2-x_1)} \begin{pmatrix} e^{-i\varphi/2} & o \\ o & e^{i\varphi/2} \end{pmatrix}$$

gives unique solutions for $\bar{k}$ and φ:

$$\bar{k} = \frac{k' + k''}{2} \tag{24}$$

$$\varphi = (k'' - k')(x_2 - x_1) \tag{25}$$

The translational and rotational phases are formally the same as in the non-relativistic case. But the expressions for k' and k" are now different:

$$\varphi(x_2-x_1) = \frac{1}{\hbar c}\left[\sqrt{(E-\mu B)^2-m^2c^2} - \sqrt{(E+\mu B)^2-m^2c^4}\right]\cdot(x_2-x_1) \tag{26}$$

To the transformation matrix $\hat{W}_{x_1 \to x_2}$ corresponds the 4x4 transformation matrix $W_{x_1 \to x_2}$ of components of the state $\Phi_{II}(x)$ II(x)
in the Cartesian basis ($\vec{e}_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$, $\vec{e}_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$, $\vec{e}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$, $\vec{e}_4 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$) defined by:

$$\Phi_{II}(x_2) = W_{x_1 \to x_2} \Phi_{II}(x_1) \tag{27}$$

By substituting into this relation the explicit expressions for U_1 and U_2 we find:

$$W_{x_1 \to x_2} = \begin{pmatrix} e^{ik'(x_2-x_1)} & & & \\ & e^{ik''(x_2-x_1)} & & \\ & & e^{ik''(x_2-x_1)} & \\ & & & e^{ik'(x_2-x_1)} \end{pmatrix} \tag{28}$$

This matrix is a product of the same factor as in $\hat{W}_{x_1 \to x_2}$ which represents the translation and of a matrix which represents the rotation, around the z-axis. This latter representation is a direct sum of two complex conjugate representations[10]

$$D^{o\frac{1}{2}}(x_2-x_1) \oplus D^{\frac{1}{2}o}(x_2-x_1) = \begin{pmatrix} e^{-i(k''-k')(x_2-x_1)/2} & & & \\ & e^{i(k''-k')(x_2-x_1)/2} & & \\ & & e^{i(k''-k')(x_2-x_1)/2} & \\ & & & e^{-i(k''-k')(x_2-x_1)/2} \end{pmatrix} \tag{29}$$

so that

$$W_{x_1 \to x_2} = e^{i\bar{k}(x_2-x_1)}\left[D^{0\frac{1}{2}}(x_2-x_1) \oplus D^{\frac{1}{2}0}(x_2-x_1)\right] \tag{30}$$

Thus group theory allows us to accomplish our first task (to write $\Phi_{II}(x)$ in the form which explicitly shows the existence of two symmetry groups). We have accomplished at the same time our second aim (to identify in $\Phi_{II}(x)$ the terms which describe the external (translational) and the internal (spin) motion).

The representation (23) of the evolution of the state (19) from the point x_1 to the point x_2 suggests that in the state $\Phi_{II}(x)$ particle moves with the momentum $\bar{p} = \hbar\bar{k}$. This motion is described by the plane wave $e^{i\bar{k}x}$. At the same time the spin precesses with the frequency Ω such that the angle of rotation along the path $L = x_2 - x_1$ equals to φ in (25)

$$\Omega t = \varphi(L) = (k'' - k')L \tag{32}$$

In order to find the frequency of precession Ω in the Laboratory frame, which corresponds to the Larmor frequency ω_L of the spin precession in the rest frame we will introduce particle velocity by:

$$\bar{p} = mV/\sqrt{1-(V/c)^2} \quad \Longleftrightarrow \quad V = \bar{p}/\sqrt{m^2+(\bar{p}/c)^2} \tag{33}$$

By substituting $t = L/V$ into (32) we find

$$\Omega = (p''-p')V/\hbar = (p''^2 - p'^2)/2\hbar\sqrt{m^2+(p'+p'')^2/4c^2} =$$

$$= -(2\mu BE/\hbar mc^2)/\sqrt{1+\left[\sqrt{p^2-(2\mu BE/c^2)+(\mu B/c)^2}+\sqrt{p^2+(2\mu BE/c^2)+(\mu B/c)^2}\right]/4m^2c^2} \tag{34}$$

Limiting cases

a) Zero mass limit

For m=0 (neutrinos) the expressions (11) for p' and p" simplify

$$p' = (E+\mu B)/c, \quad p'' = (E-\mu B)/c \tag{35}$$

Consequently the angle of rotation and precession frequency are

$$\varphi = -2\mu BL/\hbar c, \qquad \Omega = -2\mu B/\hbar \tag{36}$$

b) Ultrarelativistic limit

In the ultrarelativistic limit, characterized by

$$pc = E - m^2c^4/2E \tag{37}$$

we have:

$$p' \approx [(E+\mu B)/c] - m^2c^4/(E+\mu B)c, \quad p'' \approx [(E-\mu B)/c] - m^2c^4/(E-\mu B)c$$

$$p'' - p' = -(2\mu B/c)[1 + m^2c^4/(E^2 - \mu^2B^2)]$$

$$\bar{p} = (E/c) - (m^2c^4/Ec)/[1 - (\mu B/E)^2], \quad V = c(1 - m^2c^4/2E^2)$$

$$\Omega \approx -(2\mu B/\hbar)[1 - (m^2c^4/2E^2) + m^2c^4/(E^2 - \mu^2B^2)]$$

$$\varphi \approx -(2\mu BL/\hbar c)[1 + m^2c^4/(E^2 - \mu^2B^2)] \tag{38}$$

If moreover the magnetic field is weak the above given expressions for Ω and φ are further simplified

$$\Omega \approx -(2\mu B/\hbar)[1+(mc/E)^2/2] \tag{39}$$

$$\varphi \approx -(2\mu BL/\hbar c)[1 + (mc/E)^2]$$

c) Nonrelativistic limit

By substituting $E \approx mc^2$ in the relations

$$p'^2 = p^2+(2E\mu B/c^2)+(\mu B/c)^2, \quad p''^2 = p^2-(2E\mu B/c^2)+(\mu B/c)^2$$

and letting $c \to \infty$, the usual nonrelativistic relations appear:

$$p'^2 = p^2+2m\mu B, \qquad p''^2 = p^2-2m\mu B \tag{40}$$

Consequently

$$\bar{p} = \left[\sqrt{p^2+2m\mu B}+\sqrt{p^2-2m\mu B}\,\right]/2, \qquad V = \bar{p}/m$$

$$\Omega = (p''^2-p'^2)/2m\hbar = -2\mu B/\hbar = \omega_L$$

$$\varphi(L) = (-\sqrt{p^2+2\mu mB}+\sqrt{p^2-2m\mu B})\,L/\hbar \tag{41}$$

In the weak field limit ($\mu B \ll p^2/2m$) these expressions for $\bar{p}$, v and $\varphi(L)$ further simplify

$$\bar{p} \approx p, \qquad V \approx p/m, \qquad \varphi(L) = -2\mu BL/\hbar V = \Omega L/V \tag{42}$$

whereas Ω remains as in (41).

IV. Tunneling

In this section we consider the transmission through the magnetic potential barrier as shown in Figure 1 of a general positive energy incoming stationary wave of the form

$$\phi_I^{in}(x) = e^{ikx}(\alpha_o U_1(k,o)+\beta_o U_2(k,o)) \tag{43a}$$

We shall denote the reflected wave in region I by

$$\phi_I^{ref}(x) = e^{-ikx}(\tilde{\alpha} U_1(-k,o)+\tilde{\beta} U_2(-k,o)) \tag{43b}$$

$$\phi_I(x) = \phi_I^{in}(x)+\phi_I^{ref}(x) \tag{43c}$$

The waves in region II are

$$\phi_{II}(x) = e^{ik'x}\alpha' U_1(k',B)+e^{ik''x}\beta'' U_2(k'',B)$$

$$+e^{-ik'x}\tilde{\alpha}' U_1(-k',B)+e^{-ik''x}\tilde{\beta}'' U_2(-k'',B) \tag{43d}$$

and the transmitted wave in region III is

$$\phi_{III}(x) = e^{ikx}(\alpha U_1(k,o))+\beta U_2(k,o)) \tag{43e}$$

Note that U_1 and U_2 are functions of the corresponding k-values.

Since our wave equation is of first order, it is sufficient to impose the continuity condition at the boundaries at x=o and x=a. These are from (43) and (16) .

$$(\alpha_o+\tilde{\alpha})\sqrt{e} = (\alpha'+\tilde{\alpha}')\sqrt{e'}\sqrt{E/(E+\mu B)}$$

$$(\alpha_o-\tilde{\alpha})(k/\sqrt{e}) = (\alpha'-\tilde{\alpha}')(k'/\sqrt{e'})\sqrt{E/(E+\mu B)}$$

$$(\alpha' e^{ik'a}+\tilde{\alpha}' e^{-ik'a})\sqrt{e'}\sqrt{E/(E+\mu B)} = \alpha e^{ika}\sqrt{e}$$

$$(\alpha' e^{ik'a}-\tilde{\alpha}' e^{-ik'a})(k'/\sqrt{e'})\sqrt{E/(E+\mu B)} = \alpha e^{ika}k/\sqrt{e}$$

$$\tag{44}$$

Analogous set of equations hold for the coefficients $\tilde{\beta}, \beta'', \tilde{\beta}''$, β, coming from the U_2-component of the wave function. The former four equations can be solved for $\tilde{\alpha}$, α', $\tilde{\alpha}'$, and α in terms of the given incoming wave amplitude α_o.

The result of some lengthy algebra is:

$$\tilde{\alpha} = \alpha_o(k'^2e^2-k^2e'^2)(e^{ik'a}-e^{-ik'a})f'(k,k')$$

$$\alpha' = \alpha_o 2k\left[\sqrt{e\cdot e'(E+\mu B)/E}\right](k'e+ke')e^{-ik'a}f'(k,k')$$

$$\tilde{\alpha}' = \alpha_o 2k\left[\sqrt{e\cdot e'(E+\mu B)/E}\right](k'e-ke')e^{ik'a}f'(k,k')$$

$$\alpha = \alpha_o 4ke'k'ee^{-ika}f'(k,k') \tag{45a}$$

where

$$f'(k,k') = \left[(k'e+ke')^2e^{-ik'a}-(k'e-ke')^2e^{ik'a}\right]^{-1} \tag{46a}$$

For β's the solutions are the same with k' replaced by k", e' replaced by e" and $E+\mu B$ replaced by $E-\mu B$

$$\tilde{\beta} = \beta_o(k''^2e^2-k^2e''^2)(e^{ik''a}-e^{-ik''a})f''(k,k'')$$

$$j_1 = \bar{\psi}\gamma_1\psi = \psi^+\gamma_0\gamma_1\psi_1 = \psi^+\alpha_1\psi_1$$

$$= (\alpha^* U_1^+ + \beta^* U_2^+)\alpha_1(\alpha U_1 + \beta U_2)$$

$$j_1 = (cp/E)(|\alpha|^2 + |\beta|^2) \qquad (59)$$

In region I for the total incident and reflected wave

$$\phi_I = \phi_I^{in} + \phi_I^{ref}$$

we find for the current

$$j_1^{(I)} = (cp/E)(|\alpha_o|^2 + |\beta_o|^2) - (cp/E)(|\tilde{\alpha}|^2 + |\tilde{\beta}|^2) \qquad (60)$$

(Note-k in the reflected current).

This is indeed equal to the current in region III

$$j_1^{III} = (cp/E)(|\alpha|^2 + |\beta|^2) \qquad (61)$$

The actual solution of the problem gives a stronger condition, as we have seen, that the currents of the U_1 and U_2 - components are separately conserved

In region II we obtain for the current

$$j_1^{(II)} = [cp'/(E+\mu B)]\,(|\alpha'|^2 - |\tilde{\alpha}'|^2) + [cp''/(E-\mu B)]\,(|\beta''|^2 - |\tilde{\beta}''|^2) \qquad (62)$$

Taking into account the relations (50) one verifies the current conservation relation between the first and the second region.

$$j_1^{(I)} = j_1^{(II)}$$

It is interesting that although our particle is neutral $j_\mu = \bar{\psi}\gamma_\mu\psi$ gives the correct matter current (not the charge current).

There is a second current

$$j_\mu^{magn} = (\bar{\psi}\,\sigma_{\mu\nu}\,\psi)^{,\nu}$$

which is automatically conserved: $\partial j_\mu^{magn}/\partial x_\mu = 0$. But this current is automatically zero in our situation. The x-component is zero, because ψ only depends on x, $j_1^{magn} = (\bar{\psi}\,\sigma_{1\nu}\,\psi)^{,\nu}$. The zero component of the magnetic current

$$j_0^{magn} = \frac{\partial}{\partial x}(\bar{\psi}\,\sigma_{01}\,\psi) = i\frac{\partial}{\partial x}(\bar{\psi}\gamma_1\psi)$$

also vanishes for a pure ingoing and outgoing wave.

V. Composition law for tunneling

Elementary Reflections and Transmissions

In order to get a better physical insight into the solution (45) it is convenient, as in nonrelativistic case, to study also the tunneling through two step-like barriers shown of Figs 3 and 4, separately.

For the barrier on Fig.3. the stationary solutions are:

$$\psi_I(x) = e^{ikx}[A_oU_1(k,o) + B_oU_2(k,o)] + e^{-ikx}[\tilde{A}U_1(-k,o) + \tilde{B}U_2(-k,o)]$$

$$\psi_{II} = e^{ik'x}A'U_1(k;B) + e^{ik''x}B''U_2(k;B) \qquad (63)$$

with

$$\tilde{A} = [(ke' - k'e)/(k'e + ke')]\,e^{i2bk}A_o$$

$$A' = [2k\sqrt{ee'}/(k'e + ke')]\,\sqrt{(E+\mu B)/E}\cdot e^{ib(k-k')}\,A_o$$

$$\tilde{B} = [(ke'' - k''e)/(ke'' + k''e)]\cdot e^{i2bk}\,B_o$$

$$B'' = [2k\sqrt{ee''}/(ke'' + k''e)]\,\sqrt{(E-\mu B)/E}\,e^{ib(k-k'')}\,B_o \qquad (64)$$

By comparing initial with reflected wave and initial with transmitted wave at the point x=b we conclude that in order to identify the boundary effects it is appropriate to introduce elementary (local) reflection and transmission coefficients for the first and second component as given below:

$$R'_F = (ke'-k'e)/(ke'+k'e) \equiv R'$$

$$R''_F = (ke''-k''e)/(ke''+k''e) \equiv R''$$

$$T'_F = \left[2k\sqrt{ee'}/(ke'+k'e)\right]\sqrt{(E+\mu B)/E}$$

$$T''_F = \left[2k\sqrt{ee''}/(ke''+k''e)\right]\sqrt{(E-\mu B)/E} \qquad (65)$$

The index F denotes the transmission from vacuum into field.

For the barrier on fig.4 stationary solutions are:

$$\psi_I(x) = A_o e^{ik'x}\, U_1(k',B) + B_o e^{ik''x} U_2(k'',B)$$
$$+ Ae^{-ik'x} U_1(k',B) + B'e^{-ik''x} U_a(k'',B)$$

$$\psi_{II}(x) = Ae^{ikx} U_1(k,o) + Be^{ikx} U_2(k,o) \qquad (66)$$

where

$$A = A_o\left[2k'\sqrt{ee'}/(ke'+k'e)\right]\sqrt{E/(E+\mu B)}\, e^{ia(k'-k)}$$

$$B = B_o\left[2k''\sqrt{ee''}/(ke''+k''e)\right]\sqrt{E/(E-\mu B)}\, e^{ia(k''-k)}$$

$$\tilde{A}' = A_o\left[(k'e-ke')/(k'e+ke')\right]e^{i2ak'}$$

$$\tilde{B}' = B_o\left[(ek''-ke'')/(ek''+ke'')\right]e^{i2ak} \qquad (67)$$

The effect of the boundary between the field and the vacuum we describe through the following elementary (local) reflection and transmission coefficients:

$$R'_V = \left[(k'e-ke')/(k'e+ke')\right] = -R'$$

$$R''_V = \left[(ek''-ke'')/(k''e+ke'')\right] = -R''$$

$$T'_V = \left[2k'\sqrt{ee'}/(ke'+k'e)\right]\sqrt{E/(E+\mu B)}$$

$$T''_V = \left[2k''\sqrt{ee''}/(ke''+k''e)\right]\sqrt{E/(E-\mu B)} \qquad (68)$$

By comparing (65) and (68) we see that reflection from vacuum differs from the reflection from the field only in sign.

Reflection and Transmission matrices

In order to unify the representation of all effects which happen along the way of the neutral particle which tunnells through the magnetic field it seems appropriate to introduce for boundary effects the matrices analogous to the matrices (23) which describe the transformations inside the field.

We write the law of reflection and transmission of spin at the barrier at x = b in the $U_1 - U_2$ basis

$$\hat{\psi}_I^{refl}(b) = \hat{R}_F\, \hat{\psi}_I^{in}(b), \qquad \hat{\psi}_{II}(b) = \hat{T}_F \hat{\psi}_I^{in}(b) \qquad (69)$$

and similarly for the barrier at x = a

$$\hat{\psi}_I^{refl}(a) = \hat{R}_V \hat{\psi}_I^{in}(a) \qquad \hat{\psi}_{II}(a) = \hat{T}_V\, \hat{\psi}_I(a) \qquad (70)$$

where

$$\hat{R}_V = \begin{pmatrix} R' & 0 \\ 0 & R'' \end{pmatrix}, \quad \hat{T}_F = \begin{pmatrix} T'_F & 0 \\ 0 & T''_F \end{pmatrix}, \quad \hat{R}_V = \begin{pmatrix} -R' & 0 \\ 0 & -R'' \end{pmatrix}, \quad \hat{T}_V = \begin{pmatrix} T'_V & 0 \\ 0 & T''_V \end{pmatrix} \qquad (71)$$

Because the two components are reflected or transmitted differently the net effect at each boundary can also be expressed as an overall attenuation of the amplitude and rotation by an appropriate imaginary angle

$$\hat{R}_F = \hat{\delta}_z\, r\, D^{o\ 1/2}(\beta^r)$$

$$r = \sqrt{(ke'-k'e)(k''e-ke'')/(ke'+k'e)(k''e+ke'')}$$

$$\beta^r = i\ln\left[(ke'-k'e)(ke''+k''e)/(ke'+k'e)(k''e-ke'')\right] \qquad /72)$$

$$\hat{T}_F = t_F\, D^{o1/2}(\beta_F^t)$$

$$t_F = (T_F' T_F'')^{1/2} = 2k[e/E(ke'+k'e)(ke''+k''e)]^{1/2}[e'e''(E+\mu B)(E-\mu B)]^{1/4}$$

$$\beta_F^t = i\ln[(e'(E+\mu B))^{1/2}(ke'+k'e)/(e''(E-\mu B))^{1/2}(ke''+k''e)] \qquad (73)$$

$$\hat{R}_V = -\hat{\delta}_z\, r D^{o1/2}(\beta^r) \qquad (74)$$

$$\hat{T}_V = t_V D^{o1/2}(\beta_V^t)$$

$$t_V = (T_V' T_V'')^{1/2} = 2[k'k''eE((ke'+k'e)(ke''+k''e)]^{1/2}[e'e''/(E+\mu B)(E-\mu B)]^{1/4}$$

$$\beta_V^t = i\ln(T_V' T_V'') = i\ln[k'e'(ke''+k''e)(E-\mu B)^{1/2}/k''e''(ke'+k'e)(E+\mu B)^{1/2}] \qquad (75)$$

By comparing reflection matrices $\hat{R}_F$ and $\hat{R}_V$ we see that they differ only in sign. The angle of rotation β^r, as well as the attenuation coefficient r are the same.

Composition of Successive Reflections and Transmissions

The complicated coefficients α's and β's in eqs.(45) - specially the complex denominators - have a simple intuitive explanation as composed of the individual barrier effects of Fig.3. and Fig.4. If we expand functions $f'(k,k')$ and $f''(k,k'')$ defined in (46) into geometric series

$$f'(k,k') = \left[e^{ik'a}/(k'e+ke')^2\right]\left[1+Q'+Q'^2+\dots\right] =$$

$$= \left[e^{ik'a}/(k'e+ke')^2\right]\cdot G' \qquad (76)$$

we see that

$$Q' = \left[(k'e-ke')/(k'e+ke')\right]^2 e^{i2ak'} = R'^2 e^{i2ak'} \qquad (77)$$

is a product of two reflection coefficients from the vacuum (corresponding to the reflections of the internal wave from the boundaries at x=0 and x=a) and of an exponential phase corresponding to the propagation inside the boundary from x=0 to x=a in +x and -x directions. Thus the geometric series sum infinitely many reflections and propagations between the two barriers.

Combining the series (76), the reflection and transmission coefficients (65) and (68) at the boundaries, and the identities

$$(1-Q')^{-1} = 1 + Q'G' \qquad (78)$$

$$R'^2-1 = -T_V' T_F' \qquad (79)$$

$$\tilde{\alpha}/\alpha_o = (k'^2e^2-k^2e'^2)(e^{ik'a}-e^{-ik'a})\cdot f'(k,k') =$$

$$= R'(1-e^{2ik'a})G' = R'+R'G'e^{2ik'a}(R'^2-1) \qquad (80)$$

we can write the solution $\phi(x)$ as follows:

$$\phi_I(x) = e^{ikx}[\alpha_o U_1(k,o)+\beta_o U_2(k,o)]$$
$$+ e^{-ikx}[\alpha_o R'U_1(-k,o)+\alpha_o T_F' e^{2ik'a}(-R')T_V' G'U_1(-k,o)]$$
$$+ e^{-ikx}[\beta_o R''U_2(-k,o)+\beta_o T_F'' e^{2ik''a}(-R'')T_V'' G''U_2(-k,o)]$$

$$\phi_{II}(x) = e^{ik'x}\alpha_o T_F' G'U_1(k',B)+e^{ik''x}\beta_o T_F'' G''U_2(k'',B)$$
$$+e^{-ik'x}\alpha_o T_F'(-R')e^{2ik'a}G'U_1(-k',B)+e^{-ik''x}\beta_o T_F''(-R'')e^{2ik''a}G''U_2(-k'',B)$$

$$\phi_{III}(x) = e^{ikx}\left[\alpha_o T_F' e^{ik'a}T_V' G' e^{-ika}+\beta_o T_F'' e^{ik''a}T_V'' G'' e^{-ika}\right] \qquad (81)$$

Those expressions are completly analogous to the expresions obtained in nonrelativistic case. The interpretation of eqs. (81) is simple: The outgoing wave ϕ_{III} represents a sum of waves

passing through barriers at x=0 and at x=a once, or after any number of internal reflections. The reflected wave is a sum of waves reflected at x=0 once, or after any number of internal back and fourths. Finally, the wave inside the barrier consists of a sum of ingoing waves after one transmission and any number of internal back and fourths, and a reflected wave transmitted at x=0 reflected at x=a and again with any number of back and fourths.

Minima and maxima of reflections and transmissions

From the composition law given in (81) there follows a very simple interpretation of the conditions for the maximum and the minimum of reflection (and consequently for minimum and maximum of transmission) of components U_1 and U_2. For

$$k'\cdot a = n\pi \quad , \text{ n is an integer}$$

we have

$$Q'=R'^2, \quad G'=(1-R'^2)^{-1} \; : \; R_1=R_{1min}=0 \quad \text{and} \quad T_1=T_{1max}=1 \tag{82}$$

Further analysis shows that minimum of the reflection occurs when two terms in

$$\tilde{\alpha} = R'+R'G'e^{i2k'a}(R'^2-1)=R'+R'(R'^2-1)/(1-R'^2)=R'-R'=0 \tag{83}$$

cancel each other. This corresponds to the destructive interference of the U_1 component of the wave reflected from the boundary at x=a after any number of internal propagations and reflections. Inside the barrier two waves differing by the "optical path" 2k'a are in phase. For

$$k'a = (n+\tfrac{1}{2})\pi \quad , \text{ n is an integer}$$

$$Q'=-R'^2, G'=1/(1+R'^2) : R_1=R_{1max}=\left[2R'/(1+R'^2)\right]^2,$$

$$T_1 = T_{1min} = \left[(1-R'^2)/(1+R'^2)\right]^2 \tag{84}$$

Further analysis shows that maximum of the reflection occurs when the two terms in

$$\tilde{\alpha} = R'+R'G'e^{2ik'a}(R'^2-1) = R'+R'(1-R'^2)/(1+R'^2) \tag{85}$$

have the same phase. This corresponds to the constructive interference of the wave from the boundary at x=0 with the wave reflected from the boundary at x=a and transmitted at x=0 after any number of internal propagations and reflections. At the same time inside the barrier two waves differing by the "optical path" 2k'a are out of phase.

The usual interpretation[13] of the conditions of the maxima of the reflection coefficient in spinless case (which corresponds to the condition for maximum in the reflection of one spinor component) is based on resonances determined by the poles of α and β . It has been argued that in this case the particle spends the maximal time before going out. But this picture does not explain why after this long time it goes back and not forward. In the picture based on composition law (81) the condition for R to be a maximum is equivalent to the constructive interference of waves reflected from two boundaries and explains why the wave favours the reflection over the transmission.

For the maxima and minima of the reflection of the component U_2 we have analogous conditions

$$R_2 = R_{2min} \quad \text{when } k''a = n\pi$$

$$R_2 = R_{2max} \quad \text{when } k''a = (n+\tfrac{1}{2})\pi \tag{86}$$

It is interesting also to find the condition for which there is zero reflection for both spin components. One possibi-

lity is:

$$R_1 = 0 \wedge R_2 = 0 \Rightarrow k'a = 2\pi \wedge k''a = \pi \qquad \mu>0$$

$$2\sqrt{(E-\mu B)^2-m^2c^4} = \sqrt{(E+\mu B)^2-m^2c^4}$$

$$E = (5\mu B/3) \pm \sqrt{(4\mu B/3)^2 + m^2c^4}$$

and more generally

$$k'a = 2n\pi, \qquad k''a = (2n-1)\pi$$

$$2n\sqrt{(E-\mu B)^2-m^2c^4} = (2n-1)\sqrt{(E+\mu B)^2+m^2c^4}$$

$$E = \mu B\left[(1+\varkappa^2)/(1-\varkappa^2)\right] \pm \sqrt{(\mu B)^2 \cdot 4\varkappa^2/(1-\varkappa^2)^2+m^2c^4}, \quad \varkappa=(2n-1)/2n \qquad (87)$$

It may be possible to realize this condition experimentally.

Rotation angles and transmission coefficients in limiting cases

In Section III we found the expressions for angle and frequency of spin precession inside the field in three limiting cases: m=o, ultrarelativistic limit and nonrelativistic limit. Now we give the corresponding formulas for tunnelling.

a) Zero mass limit

Altough we can take $m = 0$ limit of all our results, it is much more expedient to go back to the initial equations. It is immediately seen that for $m = 0$ and for p in x-direction and B in z-direction, the free particle solutions *

$$U_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}, \quad U_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\1\\0\end{pmatrix}, \quad U_3 = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}, \quad U_4 = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\1\\0\end{pmatrix} \qquad (88)$$

* Helicity eigenstates which are eigenstates of the Hamiltonian are obtained from (88) with the coefficients $\alpha_0 = \beta_0 = 1/\sqrt{2}$ and $\alpha_0 = -\beta_0 = 1/\sqrt{2}$.

are also solutions in the magnetic field. But k' and k" are different $c\hbar k' = (E+\mu B)$, $c\hbar k'' = E-\mu B$, $c\hbar k = E$. Consequently there is only a spin rotation with frequency $\Omega=-2\mu B/\hbar$ for the angle $\varphi = (k''-k')a = -(2\mu Ba/\hbar c)$. There is a longitudinal Stern-Gerlach effect, but there is no reflection or attenuation of the waves. The transmission coefficients (51) are unity. This is also true, suprisingly, for $E<\mu B$. In this latter case, we must take $k''=(\mu B-E)/\hbar c$

For nontrivial reflection effects we must go to the three-dimensional case where $\vec{p}$ is not perpendicular to $\vec{B}$.

b) The ultrarelativistic limit

In terms of the two (small) parameters

$$\varepsilon = mc^2/E, \quad \eta = \mu B/E$$

eqs.(51) and (53) to lowest nonvanishing order in ε and η reduce to

$$T_1^{-1} = 1 + \eta^2\varepsilon^2 \sin^2[aE(1+\eta-\varepsilon^2/2)/\hbar c], \quad T_2^{-1}(\eta) = T_1^{-1}(-\eta)$$

$$tg\varphi' = [1 - 3\eta^2/2]\, tg[aE(1+\eta-\varepsilon^2/2)/\hbar c], \quad tg\,\varphi''(\eta) = tg\,\varphi'(-\eta)$$

Consequently:

$$\alpha \approx \alpha_0 e^{-ia\mu B/\hbar c}, \quad \beta \approx \beta_0 e^{ia\mu B/\hbar c} \qquad (89)$$

We see that the effect of the weak field boundaries on the incoming quantum ultrarelativistic particle is negligeable to order $O(\eta^2)$. The effect of the magnetic field reduces to the rotation of particle's spinor. The resultant angle of rotation $\varphi \cong -2\mu Ba/\hbar c$ depends on the length and the strength of the field.

c) Nonrelativistic limit

We found previously that for $c\to\infty$ the relations between p and p' and p and p" transform into relations (40). So we find for α and β the expressions obtained in our previuos paper[1]

$$\alpha = \alpha_o \sqrt{T_1} e^{i\varphi'} e^{-ika}, \qquad \beta = \beta_o \sqrt{T_2} e^{i\varphi''} e^{-ika} \tag{90}$$

where

$$T_1^{-1} = 1+(\mu Bm/pp')^2 \sin^2 ak', \quad T_2^{-1} = 1+(\mu Bm/pp'') \sin^2 ak''$$

$$tg\varphi' = (p^2+m\mu B/pp')\, tgak', \quad tg\varphi'' = (p^2-m\mu B/pp'')\, tgak'' \tag{91}$$

If moreover the field is weak $(\mu B<<p^2/2m)$ then $(p^2+m\mu B/pp')\simeq 1$, $(p^2-m\mu B/pp'')\simeq 1$, $(\mu Bm/pp')^2\simeq 0$, $(\mu Bm/pp'')^2\simeq 0$. Consequently:

$$\varphi' \simeq ak' \simeq a[p+(m\mu B/p)]/\hbar\,, \quad \varphi'' \simeq ak'' \simeq a[p-(m\mu B/p)]/\hbar\,, \quad \varphi(a) = \varphi'' - \varphi'' = -2am\mu B/\hbar p$$

$$T_1 \simeq 1 \qquad T_2 \simeq 1$$

$$\alpha = \alpha_o\, e^{-iam\mu B/\hbar p} = \alpha_o e^{-i\varphi(a)/2}$$

$$\beta = \beta_o\, e^{+iam\mu B/\hbar p} = \beta_o e^{i\varphi(a)/2} \tag{92}$$

Taking into account the composition law (81) we conclude that in weak field limit the boundary effects are negligeable. The effect of the field on the incoming particle reduces to the rotation of its spinor for the angle $\varphi(a) = -2m\mu Ba/p\hbar = \omega_L a/V$.

Conslusions

We have studied the eigenstates of a neutral particle with magnetic moment inside the magnetic field and the passage of relativistic neutral particles through a magnetic field barrier.

The main result inside the magnetic field is equation (20-25) showing the translational phase and spin rotation by an angle $\varphi=(k''-k')(x_2-x_1)$. The new feature in the relativistic case is the formula (34) for the precession frequency as compared with nonrelativistic formula (41).

We have also determined the transmission and reflection coefficients through a magnetic field barrier as well as the laws of spin transformations at the boundaries. We decompose the total tunnelling into multiple transmissions and reflections at the two

boundaries (81) and determine the conditions for constructive and destructive interference of incident and reflected waves. In mass zero limit there is no reflection from a magnetic barrier for the one-dimensional problem.

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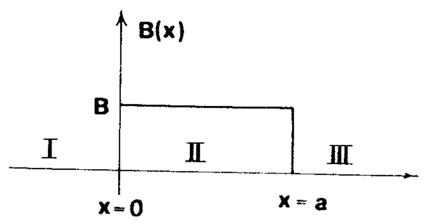
FIGURE CAPTIONS

Fig.1. **Magnetic field barrier**

Fig.2. **Regions of p^2 which lead to different combinations of imaginary and real values of p' and p'' : a) $2mc^2 > \mu B$, b) $2mc^2 < \mu B$.**

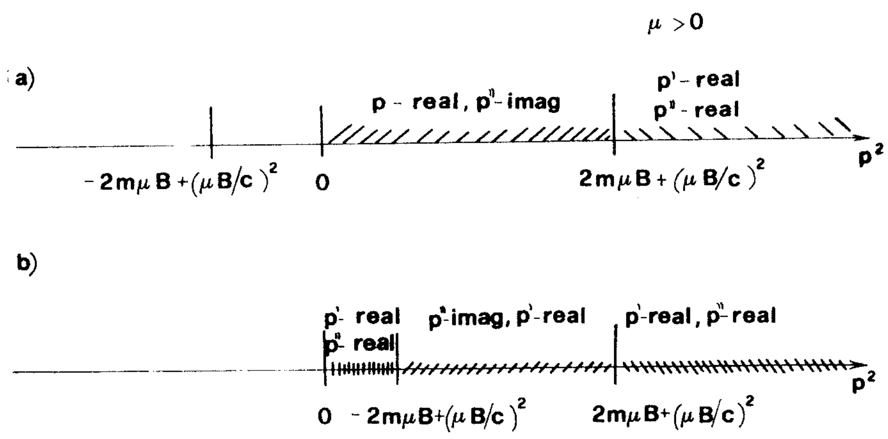
Fig.3. **Vacuum – field transition**

Fig.4. **Field – vacuum transition**



B(x)
B
I
II
III
x=0
x=a

Fig.1



μ >0
a)
p - real, p''- imag
p' - real
p'' - real
-2mμB+(μB/c)²
0
2mμB+(μB/c)²
p²
b)
p'- real
p''- real
p''-imag, p'-real
p'-real, p''-real
0
-2mμB+(μB/c)²
2mμB+(μB/c)²
p²

Fig.2

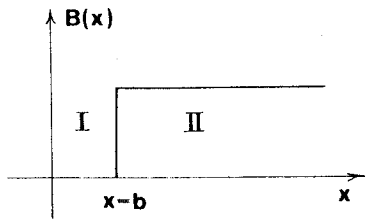


Fig. 3

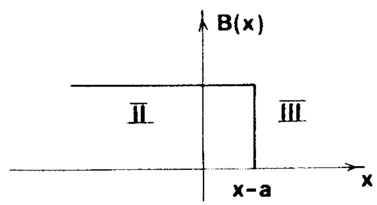


Fig.4